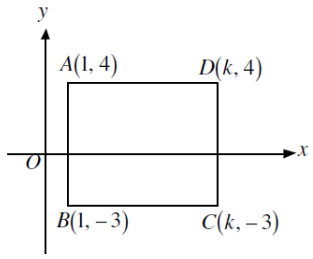
Name:_____ Date:_____

Math 10/11 Enriched: Section 7a Coordinate Geometry

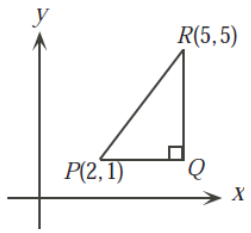
1. If the points P(0,0) , Q(0,b) and S(a,0) are the three vertices of a rectangle, then what are the coordinates of the last vertex?

2. The coordinates of one corner of a square is (3,4). The area of the square is 9. What is the coordinate of the vertex that is diagonally opposite?

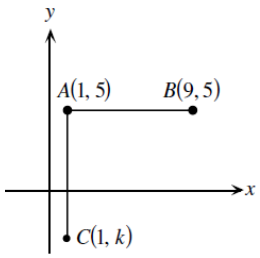
3. In the diagram, rectangle ABCD has area of 70 and "k" is positive. What is the value of "k"?



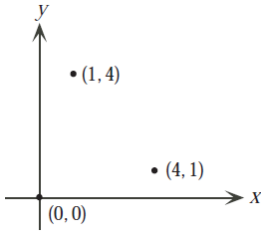
4. In the digram, PQR is a right triangle. PQ is horizontal and QR is vertical. What are the coordinates of Q?



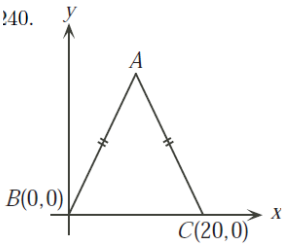
5. In the diagram, AB and AC have equal lengths, what is the value of "k"?



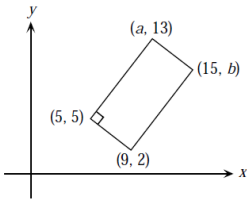
6. The coordinates of the vertices of a parallelogram as (0,0), (1,4), and (4,1). What is the area of the parallelogram?



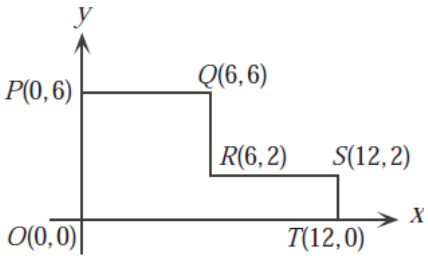
7. In the diagram, triangle ABC is isosceles and its area is 240. What is the y-coordinate of "A"?



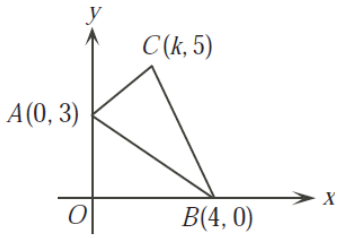
8. In the rectangle shown, what is the value of $a-b$



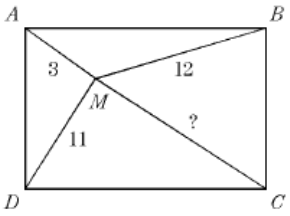
9. In the diagram below, two straight lines are to be drawn through (0,0) so that the lines divide the entire figure into 3 pieces of equal area. What are the sum of the slopes of the lines?



10. A triangle has vertices A(0,3), B(4,0) C(k,5), where 0 < k < 4. If the area of the triangle is 8, what is the value of 'K"?

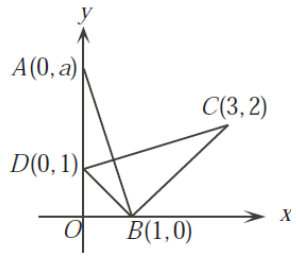


11. Given that point "M" is inside the rectangle, what is the length of MC? (Use coordinate geometry)

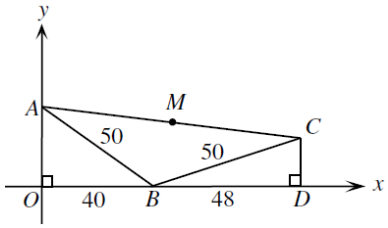


12. The distance between points A(4,0) and B is 7. If the coordinates of "B" have a sum of 7, then what are the coordinates of "B"?

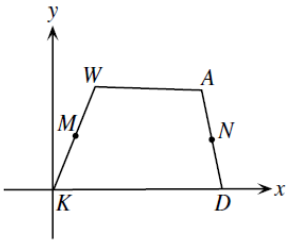
13. In the diagram below, point A(0,a) lies on the Y-axis above "D". If the triangles AOB and BCD have the same area, determine the value of "a".



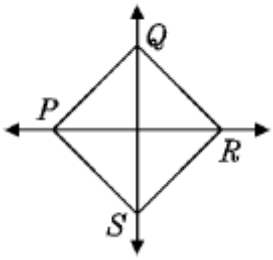
14. In the diagram, triangles AOB and CBD are right triangles and "M" is the midpoint of AC. What are the coordinates of "M"?



15. In quadrilateral KWAD, the midpoints of KW and AD are "M" and "N" respectively. If $MN = \frac{1}{2}(AW + DK)$, prove that WA is parallel to KD. EUCLID 1998 #8b

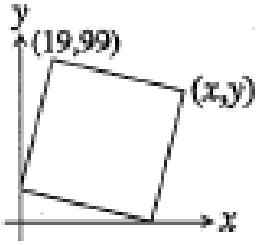


16. If the figure PQRS below is x^2, then what are the coordinates of "R" is terms of "x"?

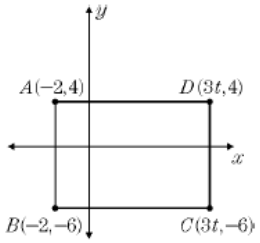


17. If the endpoints of a circle are $(-a, b)$ and $(a, -b)$, then find an expression for the area of the circle?

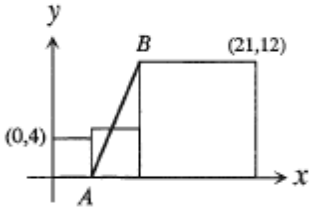
18. The square in the diagram at the right has a vertex on each coordinate axis, a vertex at (19,99), and a vertex at (x,y) as illustrated. What are the coordinates of (x,y)?



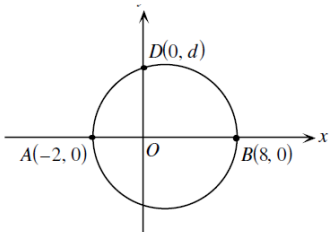
19. A point is selected at random from the interior of rectangle ABCD. Find the probability that the point selected is in quadrant II if the x-coordinate of "D" is 6times its y-coordinate.



20. Three squares are lined up along the x-axis as shown, and the points with coordinates (0,4) and (21,12) are labelled accordingly. What is the length of AB? CNML 1-3

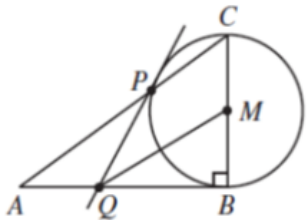


21. A circle with diameter AB as shown intersects the positive y-axis at point D(0,d). Determine the value of "d".

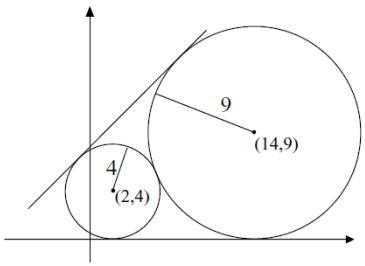


22. Each of the points $P(4,1)$, $Q(7,-8)$, and $R(10,1)$ is the midpoint of a radius of the circle "C". Determine the length of the radius of the circle "C".

23. In the diagram, triangle ABC has a right angle at "B" and "M" is the midpoint of BC. A circle is drawn using BC as its diameter. "P" is the point of intersection of the circle with AC. The tangent to the circle at "P" cuts AB at "Q". Prove that QM is parallel to AC. EUCLID



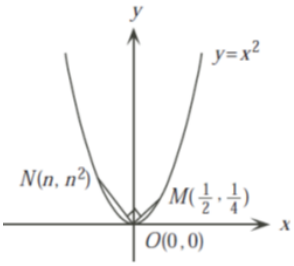
24. Circles with centers (2,4) and (14,9) have radii 4 and 9, respectively. The equation of a common external tangent to the circles can be written in the form of $y = mx + b$ with $m > 0$. What is the value of "b"? AMC #19



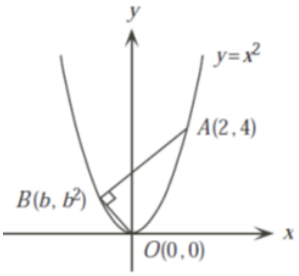
25. A parallelogram has vertices at (0,0) , (2,2) , (6,2), and (4,0). Point "P" has coordinates (1989, 1990). Line "t" passes through point "P" and divides the parallelogram into two regions of equal area. What is the slope of "t"?

CNML 1990 4-6

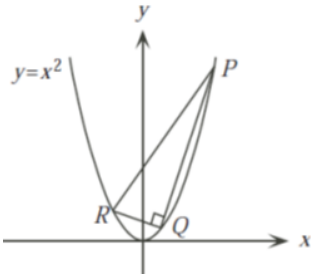
26. Points M(0.5,0.25) and N $\left(n, n^2\right)$ lie on the parabola with equation $y = x^2$. Determine the value of "n" such that $\angle MON = 90°$



27. Points A(2,4) and B $\left(b, b^2\right)$ are the endpoints of a chord of the parabola with equation $y = x^2$. Determine the value of "b" such that $\angle ABO = 90°$



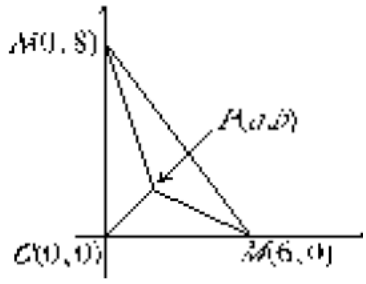
28. If the coordinates of P $\left(p, p^2\right)$, R $\left(r, r^2\right)$, and Q $\left(q, q^2\right)$, prove that $2q + p + r = 0$



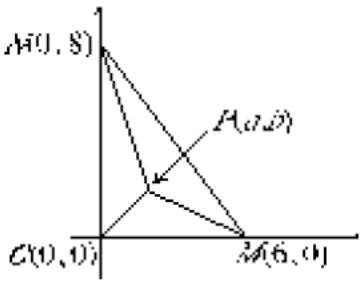
29. A piece of cheese is located at (12,10) in a coordinate plane. A mouse is at (4,-2) and is running up the line $y=-5x+18$. At the point (a,b) the mouse starts getting farther from the cheese rather than closer to it. What is the value of $a+b$? AMC 2007 12A

30. Challenge: Let R=(8,6). The lines whose equations are $8y=15x$ and $10y=3x$ contain points "P" and "Q", respectively such that "R" is the midpoint of PQ. The length PQ equals m/n, where "m" and "n' are relatively prime positive integers. Find the value of "m+n" AIME II 2001

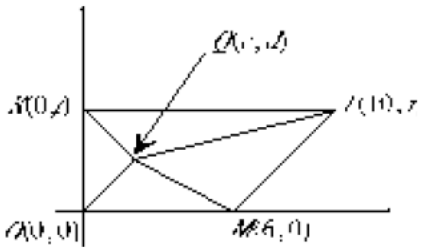
31. In the diagram triangle OMN has vertices O(0,0), M(6,0) and N(0,8). Determine the coordinates of point P(a,b) inside the triangle so that the areas of the triangles POM, PON, and PMN are all equal. Hypatia 2012



3. (a) In the diagram, triangle OMN has vertices $O(0,0)$, $M(6,0)$ and $N(0,8)$. Determine the coordinates of point $P(a,b)$ inside the triangle so that the areas of the triangles POM, PON and PMN are all equal.



(b) In the diagram, quadrilateral $OMLK$ has vertices $O(0,0)$, $M(6,0)$, $L(10,t)$, and $K(0,t)$, where $t>0$. Show that there is no point $Q(c,d)$ inside the quadrilateral so that the areas of the triangles QOM, QML, QLK, and QKO are all equal.



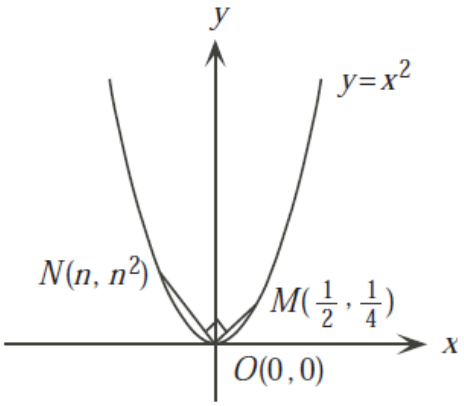
4-6. A parallelogram has vertices at (0,0), (2,2), (6,2), and (4,0). Point P has coordinates (1989,1990). Line t passes through point P and divides the parallelogram into two regions of equal area. What is the slope of t?	4-6.

Problem 4-6

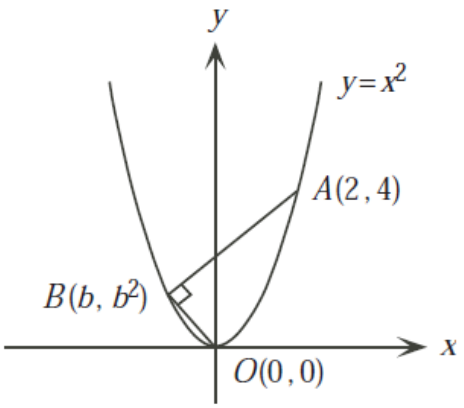
Method I: Since every parallelogram is symmetric about the intersection of its diagonals, any line which divides the parallelogram into two regions of equal area must pass through this intersection point. This point is (3,1), the midpoint of the diagonals. Since t passes through the points (3,1) and (1989,1990), the slope of t is $\frac{1989}{1986}$, or $\boxed{\frac{663}{662}}$.

Method II: The parallelogram has vertices $A(0,0)$, $B(4,0)$, $C(6,2)$, and $D(2,2)$. There must exist a point $E(x,0)$ at which line t crosses the x-axis. For the parallelogram to be split equally, the distance from $A(0,0)$ to $E(x,0)$ must equal the distance from $C(6,2)$ to $F(6-x,2)$. Since E, F, and $P(1989,1990)$ are collinear, we can solve for x.

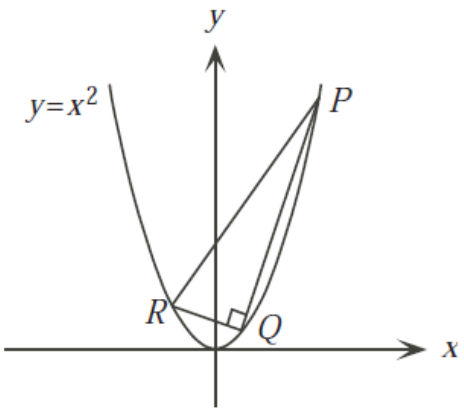
3. (a) Points $M(\frac{1}{2}, \frac{1}{4})$ and $N(n, n^2)$ lie on the parabola with equation $y = x^2$, as shown. Determine the value of n such that $\angle MON = 90°$.



(b) Points $A(2, 4)$ and $B(b, b^2)$ are the endpoints of a chord of the parabola with equation $y = x^2$, as shown. Determine the value of b so that $\angle ABO = 90°$.

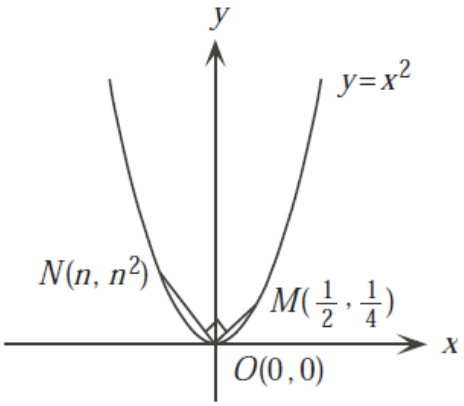


(c) Right-angled triangle PQR is inscribed in the parabola with equation $y = x^2$, as shown. Points P, Q and R have coordinates $(p, p^2), (q, q^2)$ and (r, r^2), respectively. If p, q and r are integers, show that $2q + p + r = 0$.



3. 

(a) Points $M(\frac{1}{2}, \frac{1}{4})$ and $N(n, n^2)$ lie on the parabola with equation $y = x^2$, as shown. Determine the value of n such that $\angle MON = 90°$.



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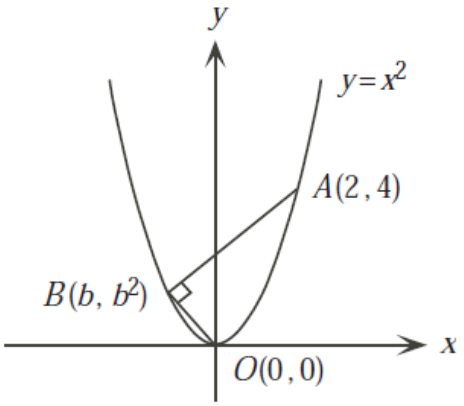
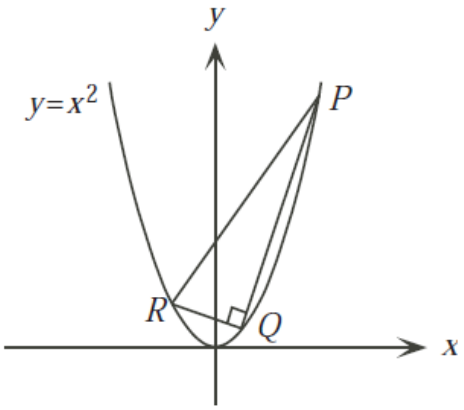


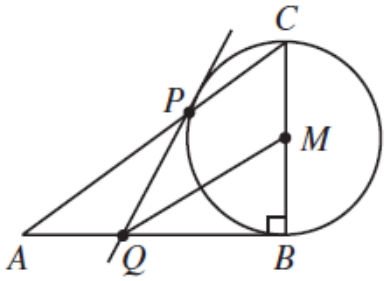
(c) Right-angled triangle PQR is inscribed in the parabola with equation $y = x^2$, as shown. Points P, Q and R have coordinates $(p, p^2), (q, q^2)$ and (r, r^2), respectively. If p, q and r are integers, show that $2q + p + r = 0$.



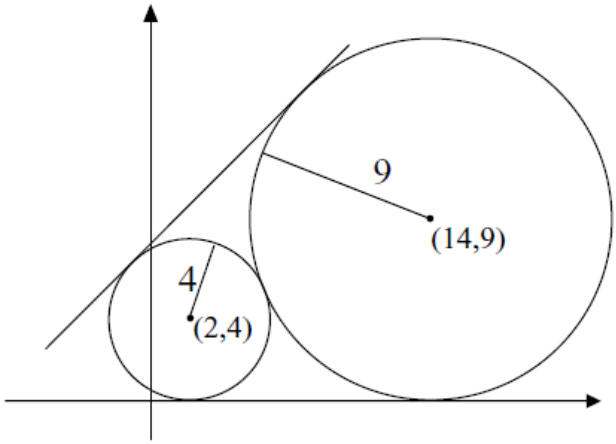
a) $a^2\pi$ b) $b^2\pi$ ∗c) $(a^2 + b^2)\pi$
d) $4a^2\pi$ e) $2b^2\pi$

7. Each of the points $P(4, 1)$, $Q(7, -8)$ and $R(10, 1)$ is the midpoint of a radius of the circle C. Determine the length of the radius of circle C.

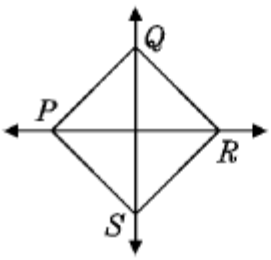
(b) In the diagram, triangle ABC has a right angle at B and M is the midpoint of BC. A circle is drawn using BC as its diameter. P is the point of intersection of the circle with AC. The tangent to the circle at P cuts AB at Q. Prove that QM is parallel to AC.



19. Circles with centers $(2, 4)$ and $(14, 9)$ have radii 4 and 9, respectively. The equation of a common external tangent to the circles can be written in the form $y = mx + b$ with $m > 0$. What is b?



(A) $\frac{908}{119}$ **(B)** $\frac{909}{119}$ **(C)** $\frac{130}{17}$ **(D)** $\frac{911}{119}$ **(E)** $\frac{912}{119}$



SAT-088.PCX

18. In the figure, the area of square $PQRS$ is x^2. What are the coordinates of R?

a) $(x, 0)$ b) $(0, x)$ c) $(0, x\sqrt{2})$

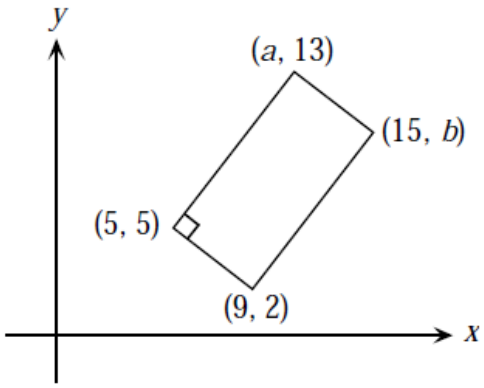
*d) $(\frac{x\sqrt{2}}{2}, 0)$ e) $\left(0, \frac{x\sqrt{2}}{2}\right)$

13. A piece of cheese is located at $(12, 10)$ in a coordinate plane. A mouse is at $(4, -2)$ and is running up the line $y = -5x + 18$. At the point (a, b) the mouse starts getting farther from the cheese rather than closer to it. What is $a + b$?

(A) 6 **(B)** 10 **(C)** 14 **(D)** 18 **(E)** 22

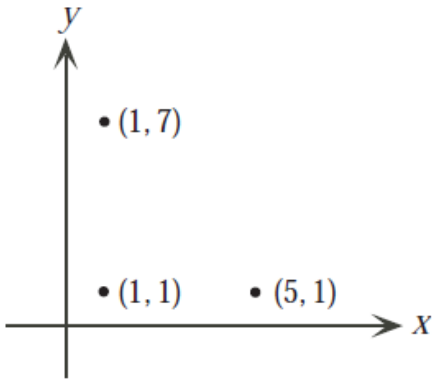
17. In the rectangle shown, the value of $a - b$ is

(A) −3 **(B)** −1 **(C)** 0
(D) 3 **(E)** 1



Points with coordinates $(1, 1)$, $(5, 1)$ and $(1, 7)$ are three vertices of a rectangle. What are the coordinates of the fourth vertex of the rectangle?

(A) $(1, 5)$ **(B)** $(5, 5)$ **(C)** $(5, 7)$
(D) $(7, 1)$ **(E)** $(7, 5)$

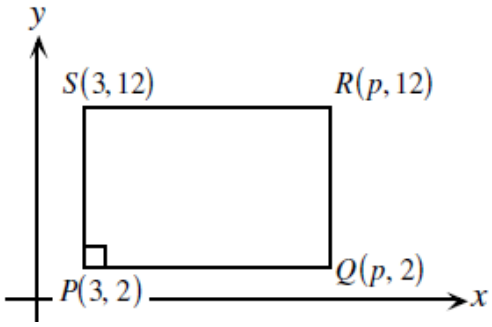


13. A piece of cheese is located at $(12, 10)$ in a coordinate plane. A mouse is at $(4, -2)$ and is running up the line $y = -5x + 18$. At the point (a, b) the mouse starts getting farther from the cheese rather than closer to it. What is $a + b$?

(A) 6 **(B)** 10 **(C)** 14 **(D)** 18 **(E)** 22

The coordinates of the vertices of rectangle $PQRS$ are given in the diagram. The area of rectangle $PQRS$ is 120. The value of p is

(A) 10 **(B)** 12 **(C)** 13
(D) 14 **(E)** 15



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15. If the points $P(0, 0)$, $Q(0, b)$, and $S(a, 0)$ are three vertices of rectangle $PQRS$, then the coordinates of point R are

*a) (a, b) b) (b, a) c) $(b, 0)$
d) $(-a, -b)$ e) $(a, -b)$

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16. The coordinates of one corner of a square are $(3, 4)$. The area of the square is 9. The coordinates of the diagonally opposite corner could be

a) $(-4, -3)$ b) $(6, 6)$ c) $(-3, 1)$
*d) $(0, 1)$ e) $(3, 1)$

4. Let $R = (8, 6)$. The lines whose equations are $8y = 15x$ and $10y = 3x$ contain points P and Q, respectively, such that R is the midpoint of $\overline{PQ}$. The length PQ equals m/n, where m and n are relatively prime positive integers. Find $m + n$.